Question 1:

Find the area of the region bounded by the curve $y^2 = x$ and the lines $x = 1$, $x = 4$ and the $x$-axis.

ANS:

The area of the region bounded by the curve $y^2 = x$, the lines $x = 1$ and $x = 4$, and the $x$-axis is the area ABCD.

\[
\text{Area of } ABCD = \int_{1}^{4} y \, dx = \int_{1}^{4} \sqrt{x} \, dx = \left[ \frac{x^{3/2}}{3/2} \right]_{1}^{4} = \frac{2}{3} \left[ (4)^{3/2} - (1)^{3/2} \right] = \frac{2}{3} [8 - 1] = \frac{14}{3} \text{ units}
\]

Question 2:

Find the area of the region bounded by $y^2 = 9x$, $x = 2$, $x = 4$ and the $x$-axis in the first quadrant.

ANS:
The area of the region bounded by the curve, \( y^2 = 9x \), \( x = 2 \), and \( x = 4 \), and the \( x \)-axis is the area \( \text{ABCD} \).

\[
\text{Area of ABCD} = \int_a^b y \, dx \\
= \int_a^b 3\sqrt{x} \, dx \\
= 3 \left[ \frac{x^{3/2}}{3/2} \right]_a^b \\
= 2 \left[ \frac{x^{3/2}}{3/2} \right]_a^b \\
= 2 \left[ (4)^{3/2} - (2)^{3/2} \right] \\
= 2 \left[ 8 - 2\sqrt{2} \right] \\
= (16 - 4\sqrt{2}) \text{ units}
\]

**Question 3:**

Find the area of the region bounded by \( x^2 = 4y \), \( y = 2 \), \( y = 4 \) and the \( y \)-axis in the first quadrant.

ANS:

\[
\text{Area of ABCD} = \int_a^b x \, dy \\
= \int_a^b \sqrt{4y} \, dy \\
= 2 \int_a^b \sqrt{y} \, dy \\
= 2 \left[ \frac{y^{3/2}}{3/2} \right]_a^b \\
= \frac{4}{3} \left[ y^{3/2} \right]_a^b \\
= \frac{4}{3} \left[ (4)^{3/2} - (2)^{3/2} \right] \\
= \frac{4}{3} \left[ 8 - 2\sqrt{2} \right] \\
= \frac{16}{3} - \frac{8\sqrt{2}}{3} \text{ units}
\]
The area of the region bounded by the curve, \( x^2 = 4y, y = 2, \) and \( y = 4, \) and the \( y\)-axis is the area \( AB\). CD.

\[
\text{Area of } AB\ CD = \int_2^4 \frac{\sqrt{y}}{2} \, dy \\
= \frac{3}{2} \int_2^4 \sqrt{y} \, dy \\
= \frac{3}{2} \left[ \frac{y^{3/2}}{3/2} \right]_2^4 \\
= \frac{3}{2} \left[ \frac{4^{3/2}}{3/2} - \frac{2^{3/2}}{3/2} \right] \\
= \frac{3}{2} \left[ \frac{8 - 2\sqrt{2}}{3} \right] \\
= \frac{3}{2} \left[ \frac{32 - 8\sqrt{2}}{3} \right] \text{ units}
\]

Question 4:

Find the area of the region bounded by the ellipse \( \frac{x^2}{16} + \frac{y^2}{9} = 1 \)

**ANS:**

It can be observed that the ellipse is symmetrical about \( x\)-axis and \( y\)-axis.

\[
\therefore \text{Area bounded by ellipse} = 4 \times \text{Area of } OAB
\]

\[
\text{Area of } OAB = \int_0^1 y \, dx \\
= \int_0^1 \frac{1 - \frac{x^2}{16}}{2} \, dx \\
= \frac{2}{3} \int_0^1 \sqrt{16 - x^2} \, dx \\
= \frac{2}{3} \left[ \frac{x}{2} \sqrt{16 - x^2} + \frac{16}{2} \sin^{-1} \frac{x}{4} \right]_0^1 \\
= \frac{2}{3} \left[ \frac{8\pi}{2} + 8\sin^{-1}(1) - 0 - 8\sin^{-1}(0) \right] \\
= \frac{3}{4} \left[ 8\pi \right] \\
= \frac{3}{4} [4\pi] \\
= 3\pi
\]

Therefore, area bounded by the ellipse \( = 4 \times 3\pi = 12\pi \) units
Find the area of the region bounded by the ellipse \( \frac{x^2}{4} + \frac{y^2}{9} = 1 \)

**ANS:**

\[
\frac{x^2}{4} + \frac{y^2}{9} = 1
\]

\[\Rightarrow y = 3\sqrt{1 - \frac{x^2}{4}} \quad \text{...(1)}\]

It can be observed that the ellipse is symmetrical about x-axis and y-axis.

\[\therefore \text{Area bounded by ellipse} = 4 \times \text{Area OAB}\]

\[\therefore \text{Area of OAB} = \int_0^2 y \, dx \]

\[= \int_0^2 3\sqrt{1 - \frac{x^2}{4}} \, dx \quad \text{[Using (1)]}\]

\[= \frac{3}{2} \int_0^2 \sqrt{4 - x^2} \, dx\]

\[= \frac{3}{2} \left[ \frac{x}{2} \sqrt{4 - x^2} + \frac{4}{2} \sin^{-1} \frac{x}{2} \right]_0^2\]

\[= \frac{3}{2} \left[ \frac{2\pi}{2} \right]\]

\[= \frac{3\pi}{2}\]

Therefore, area bounded by the ellipse = \( 4 \times \frac{3\pi}{2} = 6\pi \, \text{units} \)
Find the area of the region in the first quadrant enclosed by x-axis, line \( x = \sqrt{3}y \) and the circle \( x^2 + y^2 = 4 \).

**ANS:**

The point of intersection of the line and the circle in the first quadrant is \((\sqrt{3},1)\).

Area \( OAB = \text{Area } \triangle OCA + \text{Area } ACB \)

Area of \( OAC = \frac{1}{2} \times OC \times AC = \frac{1}{2} \times \sqrt{3} \times 1 = \frac{\sqrt{3}}{2} \) \hspace{1cm} \text{...(1)}

Area of \( ABC = \int_{\sqrt{3}}^{2} y \, dx \)

\[
= \int_{\sqrt{3}}^{2} \sqrt{4-x^2} \, dx \\
= \left[ \frac{x}{2} \sqrt{4-x^2} + \frac{4}{2} \sin^{-1} \frac{x}{2} \right]_{\sqrt{3}}^{2} \\
= \left[ 2 \times \frac{\pi}{2} - \sqrt{3} \sqrt{4-3} - 2 \sin^{-1} \left( \frac{\sqrt{3}}{2} \right) \right] \\
= \left[ \pi - \frac{\sqrt{3}}{2} - 2 \left( \frac{\pi}{2} \right) \right] \\
= \left[ \pi - \frac{\sqrt{3}}{2} - \frac{2\pi}{3} \right] \\
= \left[ \frac{\pi}{2} - \frac{\sqrt{3}}{2} \right] \hspace{1cm} \text{...(2)}
\]

Therefore, area enclosed by x-axis, the line \( x = \sqrt{3}y \), and the circle \( x^2 + y^2 = 4 \) in the first quadrant = \( \frac{\sqrt{3}}{2} + \frac{3\sqrt{\pi}}{2} \) units.
Find the area of the smaller part of the circle $x^2 + y^2 = a^2$ cut off by the line $x = \frac{a}{\sqrt{2}}$

**ANS:**

The area of the smaller part of the circle, $x^2 + y^2 = a^2$, cut off by the line, $x = \frac{a}{\sqrt{2}}$, is the area $ABCD$.

It can be observed that the area $ABCD$ is symmetrical about the $x$-axis.

\[\therefore \text{Area } ABCD = 2 \times \text{Area } ABC\]

**Area of } ABC = \int_{\frac{a}{\sqrt{2}}}^{a} y \, dx\]

\[= \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sqrt{a^2 - x^2} \, dx\]

\[= \left[ \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} \right]_{\frac{a}{\sqrt{2}}}^{a}\]

\[= \left[ \frac{a^2}{2} \left( \frac{\pi}{2} \right) - \frac{a}{2\sqrt{2}} \sqrt{a^2 - \frac{a^2}{2} - \frac{a^2}{2} \sin^{-1} \left( \frac{1}{\sqrt{2}} \right)} \right] - \left[ \frac{a^2}{4} - \frac{a}{2\sqrt{2}} \sqrt{a^2 - \frac{a^2}{2} - \frac{a^2}{2} \left( \frac{\pi}{4} \right)} \right]\]

\[= \frac{a^2\pi}{4} - \frac{a^2}{4} - \frac{a^2}{8}\]

\[= \frac{a^2}{4} \left[ \pi - 1 - \pi \right]\]

\[= \frac{a^2}{4} \left[ \frac{\pi - 1}{2} \right]\]

\[\Rightarrow \text{Area } ABCD = 2 \left[ \frac{a^2}{4} \left( \frac{\pi - 1}{2} \right) \right] = \frac{a^2}{2} \left( \frac{\pi}{2} - 1 \right)\]

Therefore, the area of the smaller part of the circle, $x^2 + y^2 = a^2$, cut off by the line, $x = \frac{a}{\sqrt{2}}$, is $\frac{a^2}{2} \left( \frac{\pi}{2} - 1 \right)$ units.
Question 8:

The area between \( x = y^2 \) and \( x = 4 \) is divided into two equal parts by the line \( x = a \), find the value of \( a \).

ANS:

The line, \( x = a \), divides the area bounded by the parabola and \( x = 4 \) into two equal parts.

\[ \therefore \text{Area } \triangle OAD = \text{Area } \square ABCD \]

It can be observed that the given area is symmetrical about \( x \)-axis.

\[ \Rightarrow \text{Area } OED = \text{Area } EFCD \]

Area \( OED = \int_0^a y \, dx \)

\[ = \int_0^a \sqrt{x} \, dx \]

\[ = \left[ \frac{3}{2} x^{\frac{3}{2}} \right]_0^a \]

\[ = \frac{2}{3} (a)^{\frac{3}{2}} \quad \text{...(1)} \]
Area of $\text{EFCD} = \int_0^a \sqrt{x} \, dx$

\[
= \left[ \frac{3}{2} \cdot \frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right]_0^a
\]
\[= \frac{2}{3} \left[ 8 - \left(\frac{3}{2}\right)^2 \right] \quad \cdots (2)
\]

From (1) and (2), we obtain

\[
\frac{2}{3} (a)^{\frac{3}{2}} = \frac{2}{3} \left[ 8 - (a)^{\frac{3}{2}} \right]
\]

\[
\Rightarrow 2 \cdot (a)^{\frac{3}{2}} = 8
\]

\[
\Rightarrow (a)^{\frac{3}{2}} = 4
\]

\[
\Rightarrow a = \left(4\right)^{\frac{2}{3}}
\]

Therefore, the value of $a$ is $\left(4\right)^{\frac{2}{3}}$.

**Question 9:**

Find the area of the region bounded by the parabola $y = x^2$ and $y = |x|$

**ANS:**

The area bounded by the parabola, $x^2 = y$, and the line, $y = |x|$, can be represented as
The given area is symmetrical about y-axis.

\[ \text{\because Area } OACO = \text{Area } ODBO \]

The point of intersection of parabola, \( x^2 = y \), and line, \( y = x \), is \( A(1, 1) \).

Area of \( OACO = \text{Area } \Delta OAB - \text{Area } OBACO \)

\[ \text{\because Area of } \Delta OAB = \frac{1}{2} \times OB \times AB = \frac{1}{2} \times 1 \times 1 = \frac{1}{2} \]

Area of \( OBACO = \int_{0}^{1} y \, dx = \int_{0}^{1} x^2 \, dx = \left[ \frac{x^3}{3} \right]_{0}^{1} = \frac{1}{3} \]

\[ \Rightarrow \text{Area of } OACO = \text{Area of } \Delta OAB - \text{Area of } OBACO \]

\[ = \frac{1}{2} - \frac{1}{3} = \frac{1}{6} \]

Therefore, required area = \( 2 \left[ \frac{1}{6} \right] = \frac{1}{3} \) units

**Question 10:**

Find the area bounded by the curve \( x^2 = 4y \) and the line \( x = 4y - 2 \)

ANS:

The area bounded by the curve, \( x^2 = 4y \), and line, \( x = 4y - 2 \), is represented by the shaded area \( OBAO \).
Let A and B be the points of intersection of the line and parabola.

Coordinates of point A are \((-1, \frac{1}{4})\).

Coordinates of point B are (2, 1).

We draw AL and BM perpendicular to x-axis.

It can be observed that,

\[
\text{Area OBAO} = \text{Area OBCO} + \text{Area OACO} \quad \ldots (1)
\]

Then, Area OBCO = Area OMBC – Area OMBO

\[
\begin{align*}
&= \int_{0}^{3} \frac{x^2 + 2x}{4} \, dx - \int_{0}^{3} \frac{x^2}{4} \, dx \\
&= \frac{1}{4} \left[ \frac{x^3}{2} + 2x \right]_{0}^{3} - \frac{1}{4} \left[ \frac{x^3}{3} \right]_{0}^{3} \\
&= \frac{1}{4} \left[ 2 + 4 \right] - \frac{1}{4} \left[ \frac{8}{3} \right] \\
&= \frac{3}{2} - \frac{2}{3} \\
&= \frac{5}{6}
\end{align*}
\]

Similarly, Area OACO = Area OLAC – Area OLAO

\[
\begin{align*}
&= \int_{-1}^{0} \frac{x^2 + 2x}{4} \, dx - \int_{-1}^{0} \frac{x^2}{4} \, dx \\
&= \frac{1}{4} \left[ \frac{x^3}{2} + 2x \right]_{-1}^{0} - \frac{1}{4} \left[ \frac{x^3}{3} \right]_{-1}^{0} \\
&= \frac{1}{4} \left[ \frac{(-1)^3}{2} + 2(-1) \right] - \frac{1}{4} \left[ \frac{(-1)^3}{3} \right] \\
&= \frac{1}{4} \left[ \frac{-1}{2} - 2 \right] - \frac{1}{12} \\
&= \frac{1}{8} - \frac{1}{12} \\
&= \frac{7}{24}
\end{align*}
\]

Therefore, required area = \(\frac{5}{6} + \frac{7}{24} = \frac{9}{8}\) units
Find the area of the region bounded by the curve $y^2 = 4x$ and the line $x = 3$

**ANS:**

The region bounded by the parabola, $y^2 = 4x$, and the line, $x = 3$, is the area $OACO$.

The area $OACO$ is symmetrical about $x$-axis.

\[ \therefore \text{Area of } OACO = 2 \times \text{Area of } OAB \]

\[
\text{Area } OACO = 2 \left[ \int_{0}^{3} y \, dx \right] = 2 \int_{0}^{3} 2\sqrt{x} \, dx = 4 \int_{0}^{3} \sqrt{x} \, dx = 4 \left[ \frac{x^{3/2}}{3/2} \right]_{0}^{3} = \frac{8}{3} \left[ (3)^{3/2} \right] = 8\sqrt{3}
\]

Therefore, the required area is $8\sqrt{3}$ units.
Area lying in the first quadrant and bounded by the circle $x^2 + y^2 = 4$ and the lines $x = 0$ and $x = 2$ is

A. $\pi$

B. $\frac{\pi}{2}$

C. $\frac{\pi}{3}$

D. $\frac{\pi}{4}$

ANS:

The area bounded by the circle and the lines, $x = 0$ and $x = 2$, in the first quadrant is represented as:

\[
\therefore \text{Area } OAB = \int_{0}^{2} y \, dx = \int_{0}^{2} \sqrt{4 - x^2} \, dx = \left[ \frac{x}{2} \sqrt{4 - x^2} + \frac{4}{2} \sin^{-1} \frac{x}{2} \right]_{0}^{2} = 2 \left( \frac{\pi}{2} \right) = \pi \text{ units}
\]

Thus, the correct answer is A.
Area of the region bounded by the curve $y^2 = 4x$, $y$-axis and the line $y = 3$ is

A. 2
B. $\frac{9}{4}$
C. $\frac{9}{3}$
D. $\frac{9}{2}$

**ANS:**

The area bounded by the curve, $y^2 = 4x$, $y$-axis, and $y = 3$ is represented as

\[
\therefore \text{Area } OAB = \int_0^3 x \, dy \\
= \int_0^3 \frac{y^2}{4} \, dy \\
= \frac{1}{4} \left[ \frac{y^3}{3} \right]_0^3 \\
= \frac{1}{4} \left( \frac{27}{3} \right) \\
= \frac{9}{4} \text{ units}
\]

Thus, the correct answer is B.